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*Quis dubitet hominem
coniungere caelo?*

a cura di
Elio Antonello

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Solar and cosmological symbolism and astronomical orientations of Romanesque churches in Tuscany

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Abstract. Romanesque art and architecture are pervaded of solar symbols: for example, in Christian churches, the apse represents the spiritual fulcrum of the building and its sacredness is often emphasized by a peculiar illumination. Although many studies indicate that Romanesque churches were built with specific orientations, conferring a symbolic connotation to the building itself, a systematic work on the orientation of Romanesque churches in the ancient dioceses of Tuscany is still lacking.

In this paper, we report the study on the orientation of 383 Romanesque churches located in the territory of Tuscany based on their importance and on the conservation of original architectural elements. The detailed analysis of the location of churches showed that most of them are directed toward the sunrise. Furthermore, they can be categorized into three main groups based on a specific orientation: 1) toward the solstices, 2) toward the local equinoctial sunrise or the sunrise of the Easter period 3) towards the East-southeast and the East-northeast (*Versus Solem Orientem*).

Our investigation proposes a solar symbolism associated with the orientation of the churches we examined. In the first group, the position of the sacred buildings suggests a connection with the solstices: these events were considered by the ancient to be the heaven's gates, favourable astronomical moments for the descent of the angels of God to Earth. In the second group, the orientation is probably linked with the Easter Sunday or there might be a connection with the *Sol Aequinoctialis*, whose rays illuminated the *Sancta Sanctorum* of the Solomon's Temple, the holy temple in ancient Jerusalem. Finally, in the third group the direction towards the local sunrise, *Versus Solem Orientem* suggests a link with the direction of prayer, towards Jerusalem, which was a very common practice in medieval architecture.

1. Introduction: building churches in connection with the sky

During the Council of Nicaea, in 325 A.D., one of the main issues that were discussed was the precise determination of the vernal equinox, which was essential to establish the date of Easter. Furthermore, it was affirmed that it was essential for the devotees to face the sunrise – *Versus Solem Orientem*, which symbolized the Light of Jesus Christ – while praying. *Ecclesiarum situs plerumque talis erat: ut Fideles facie ad Altare versa orantes orientem solem, symbolum Christi - Qui est sol justitiae et lux mundi - intererentur*¹, was declared by the Council (Kozma de Papi 1861).

This idea was reaffirmed in the XI century by Gerbert of Aurillac, a scholar and teacher who, in 999 A.D., was made Pope and took the name of Sylvester II. Gerbert studied mathematics, geometry and astronomy in Spain, which in that period was under the Arab domination, and his interest in astronomy was so profound that he reintroduced to Europe the armillary sphere (Schulman J. K. 2002). It is then little wonder that, once he became the Pope, he recommended that the churches were oriented with the apse facing the *Sol Aequinoctialis*: this was a ritual itself, because the goal of it was to create a connection between the cosmos and the Earth, that is God and the man (Davy 1988; Incerti 1999; Gaspani 2000). In the XII century, theologian Johannes Belethus claimed the importance of building the churches towards the equinoctial sunrise rather than the summer solstitial sunrise: *quoque necessarium est ut aedificetur versus orientem, hoc est versus solis ortum aequinoctialem; nec verso contra aestivum solstitium, ut nonnulli et volunt et faciunt*² (Belethus 1559). Furthermore, in XIII century, Bishop Durand Guillaume reiterated the importance of the orientation towards the equinoxes: *Debet quoque (ecclesia) sic fundari, ut caput inspiciat versus Orientem... videlicet versum ortum solis, ad denotandum, quod ecclesia quae in terris militat, temperare se debet aequanimiter in prosperis, et in adversis, et not versus solstitialem, ut faciunt quidam*³, he wrote (Durand 1672).

¹ “Churches were positioned in such a way that worshippers faced the altar and the sunrise to the East and that symbolize the Christ – Sun of Justice and Light of the World”.

² “It is necessary to build (the churches) towards the East, that is the equinoctial sunrise, and not towards the summer solstice, as some claims and do”.

³ “(A church) must be built so that its head points towards the East... that is towards the sunrise, to emphasize that the Church, militant on the Earth, has to be moderate in prosperities and in adversities, and not towards the solstice, as some are built”.

These fragments highlight the fact that during the Romanesque period, albeit some churches were built with a solstitial orientation, sacred buildings were usually erected having their axis on the East-West direction. In particular, the apse was usually oriented towards the East (*Versus Solem Orientem*) so that the façade was directed to the sunset. In Christian churches, the apse represents the spiritual fulcrum of the building, the very place where the man meets God, and its sacredness is often emphasized by a peculiar solar illumination (Guènon 1977). Many important symbols are associated with the orientation towards the sunrise of the vernal equinox (*Sol Aequinoctialis*). For example, the sunrise alludes to Jesus Christ, the true source of all life, as reported by the Gospels, and in particular the sunrise of the Easter period symbolizes the resurrection of Christ.

In Romanesque churches where the original windows are intact, it is still possible to see the light effects that were once envisioned to represent the relationship between God and man, and many studies indicate that Romanesque churches were built with specific orientations, conferring a symbolic connotation to the building itself (Incerti 2015; Spinazzè 2015). Nevertheless, a systematic work on the orientation of Romanesque churches in the ancient dioceses of Tuscany is still lacking. For this study we selected 383 Romanesque churches amongst the thousands in Tuscany based on the importance of the churches and on the preservation of the original windows to examine their orientation and the solar symbolism that permeate the sacred buildings

2. Astronomical Survey

According to the “*Rationes decimarum italiae*” nei secoli XIII e XIV. *Tuscia*, a book published by the Vatican Apostolic Library, there are thousands of Romanesque churches in Tuscany (Giusti, Guidi 1932). Based on the importance of the churches and the preservation of the original windows we studied 383 churches (Salmi 1958; Bartolini 2017) that have been georeferenced on the 1:25000 map of the Italian Military Geographic Institute (Istituto Geografico Militare Italiano, IGMI).

Using the coordinates UTM-ETRF89⁴ we were able to recognize and map on Google Earth all of the churches we examined; we then measured the azimuth on the top of the roof of each church. To determine if the

⁴ UTM-ETRF89 is a geodetic reference frame. UTM: Universal Transverse of Mercator; ETRF89: European Terrestrial Reference Frame 1989.

orientation of a church was related to the sunrise it is necessary to measure the elevation angle of the local horizon. However, since those measurements are difficult to obtain, because churches are often surrounded by trees or other buildings in the area near the apse, we utilized Google Earth to identify the elevation angle profile on the prolongation of the line of the azimuth of the church.

We then directly measured the orientation of circa half of the churches (198) and, if possible, we measured also the elevation angle of the local horizon. In order to do that we determined the plane coordinates UTM-ETRF89 of at least two corners of each building or two points on the outer walls or on the façade. Coordinates were measured using a Trimble Geo7X handheld device equipped with a laser distance-meter, which is indispensable to determine the points mentioned above. For each point we performed two sets of ten GNSS⁵ measurements each that were processed and corrected using the GPS Pathfinder Office Software (TRIMBLE) and the ITALPOS network stations⁶.

Coordinates have been determined with a precision of ± 0.15 m in 70% of the cases, ± 0.30 m in 20% and ± 0.50 m in 10%. To that are associated the following angular precisions: $\pm 0.5^\circ$ in 70% of the cases, $\pm 1^\circ$ in 20% and $\pm 2^\circ$ in 10% of the cases. Data refers to an average length of a wall of 25 m. We then compared the values of the azimuth we measured with those of Google Earth: the mean difference is 1.0° , and the maximal difference is 3° . Table 1 shows the comparison between the two sets of measurements. Maximal differences between the measurements were observed in mountainous regions where high-resolution pictures are not available on Google Earth.

Table 1

Diff. $\leq 0.5^\circ$	$0.5^\circ < \text{Diff.} \leq 1^\circ$	$1^\circ < \text{Diff.} \leq 1.5^\circ$	$1.5^\circ < \text{Diff.} \leq 2^\circ$	$2^\circ < \text{Diff.} \leq 3^\circ$
66 churches	50 churches	40 churches	26 churches	16 churches

Based on these results we considered Google Earth a reliable tool for the measurements of the azimuth, especially for those churches that are

⁵ GNSS is the global navigation satellite system based on the US system NAVSTAR (GPS), the Russian system GLONASS, the European system GALILEO and the Chinese system (Beidou)

⁶ ItalPoS is an Italian GNSS network of permanent stations that delivers satellite positioning services

surrounded by many buildings (i.e. the churches located in the town centre). In fact, an enclosed space, such as the centre of towns (Florence, Prato, Pistoia, Lucca, Pisa, Siena, Arezzo, and peripheries) causes a reduction in the number of GNSS satellites that are in direct line of sight of the receiver, thereby reducing the precision of the coordinate measurement.

Based on the azimuth, the elevation angle profile and the latitude data of the churches we calculated the declination (δ)⁷ that we utilized to determine the day of the local sunrise when the Sun aligns with the church axis.

3. Data analysis: orientation of churches and solar symbolism

Because Romanesque architecture is associated with a vast solar symbolism, the analysis of the measurements we performed has to take into consideration the significant astronomic moments throughout the year, in particular the equinoxes, the solstices and the sunrise of the Easter period. It is important to note that in the X-XIII centuries the Julian calendar was still in effect. This calendar was developed based on the measurement of the duration of the tropical year carried out by Sosigenes of Alexandria in 45 BC. Despite the minimal discrepancy in the length of the duration of the year between the Julian calendar (365 days and 6 hours) and the tropical year (365 days, 5 hours, 48 minutes and 46 seconds), the difference caused a divergence of one day every 128 years. Due to this imprecision, in 325 AD, when the Council of Nicaea was held, there was already a difference of about 3 days with the tropical year. Therefore, the date of the equinox established by the Julian calendar (March 25) actually coincided with March 21. The Council then decided to fix the problem and decreed March 21 as the vernal equinox. This was clearly a temporary adjustment, because the inexactness of the calendar persisted.

In the XI century, there was a discrepancy of 6 days between the Julian calendar and the tropical year: therefore the actual day of the astronomical equinox was March 15 of the Julian calendar – and not March 21, the so called “ecclesiastical equinox” (Ximenes 1757). However, Easter was still calculated according to the Council of Nicaea: it had to be celebrated on the first Sunday after the first full moon occurring on or after the vernal

⁷ Declination (δ) was calculated as it follows: $\sin \delta = \sin \phi \cdot \sin h + \cos \phi \cdot \cos h \cdot \cos Az$ (ϕ latitude, h elevation angle, and Az azimuth). Elevation angle measurements were corrected for the atmospheric refraction using the Garfinkel algorithm.

equinox. Easter was then a Sunday comprised in the period between March 22 and April 25 of the Julian calendar.

In order to calculate the azimuth of the sunrise during the Easter period of the first half of the XI century, it is then necessary to consider this 6 days discrepancy: we should consider the Easter period as comprised in the time span between March 28 and May 1 of the Gregorian calendar. During this period the declination of the Sun is comprised between 3° and 15° and the azimuth of the sunrise is comprised between 87° (March 28) and 69.5° (May 1), with elevation angle = 0° on the horizon.

Conversely, the 6 days discrepancy has little effect on the solstices: in fact, the difference between of the azimuth of the solstitial sunrise of June 15 and December 15 (the astronomical solstices of XI century) and the azimuth of the solstices of the Julian calendar is only $7'$ of arc.

The analysis of the azimuth values revealed that 323 churches of the 383 examined (84%) are oriented towards a sector of the horizon comprised between the sunrise of the two solstices. However, if we take into consideration the declination, rather than the azimuth, 310 are the churches that are oriented towards the local sunrise (81%). There is no doubt, nevertheless, that the criterion *Versus Solem Orientem* has been widely adopted until the XIII century. Furthermore, we analysed in detail the 310 churches that are oriented towards the sunrise and we made these observations:

- 10 churches (3.2%)⁸ are oriented towards the sunrise of summer solstice (*Aestivum Solstitium*) (Figure 1);
- 19 churches (6.1%) are oriented towards the East-northeast (*Versus Solem Orientem*);
- 100 churches (32.2%) are oriented towards the sunrise of the Easter period (X-XIII century) (Figure 2). In particular, seven churches are directed towards the sunrise of March 21 of the XI century (the “ecclesiastical equinox”);
- 22 churches (7%) are oriented towards the local equinoctial sunrise (*Sol Aequinoctialis*) (Figure 3);
- 144 churches (46.5%) are oriented towards East-southeast (*Versus Solem Orientem*) (Figure 4 and 5);
- 15 churches (5%) are oriented towards or in the vicinity of the sunrise of the winter solstice (*Sol Invictus*) (Figure 6).

⁸ Percentages are referred to the 310 churches oriented towards the local sunrise.



Fig. 1. The sunlight of the summer solstice sunrise enters from the windows of the apse of the Pieve di San Pietro in Romena (Pratovecchio, Arezzo), azimuth = 57.2° , and illuminates the counter-façade (picture: June 20, 5.35 a.m.).

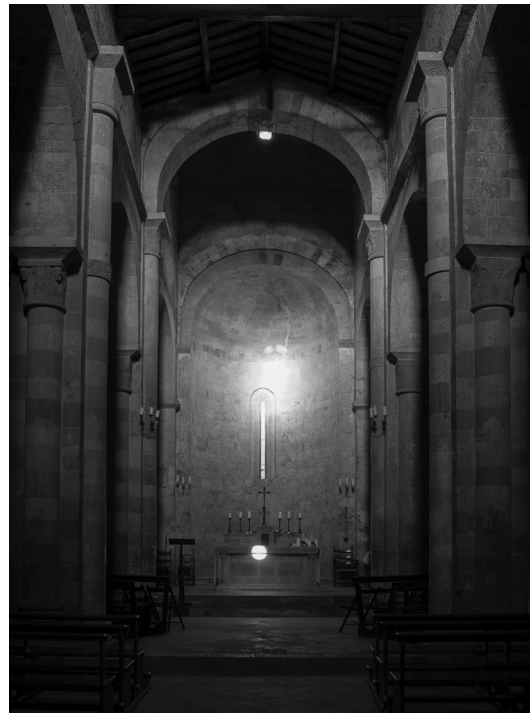


Fig. 2. The sunlight enters from the circular opening above the main entrance of the façade in the Pieve di San Giovanni Battista a Ponte allo Spino (Sovicille, Siena), azimuth = 88.3° , and illuminates the altar during the Easter period (picture: March 28, 6 p.m.).



Fig. 3. The Abbey of San Galgano (Chiusdino, Siena) is perfectly aligned to the equinoctial sunrise, azimuth = 92° (picture: March 21, 6.15 a.m.).

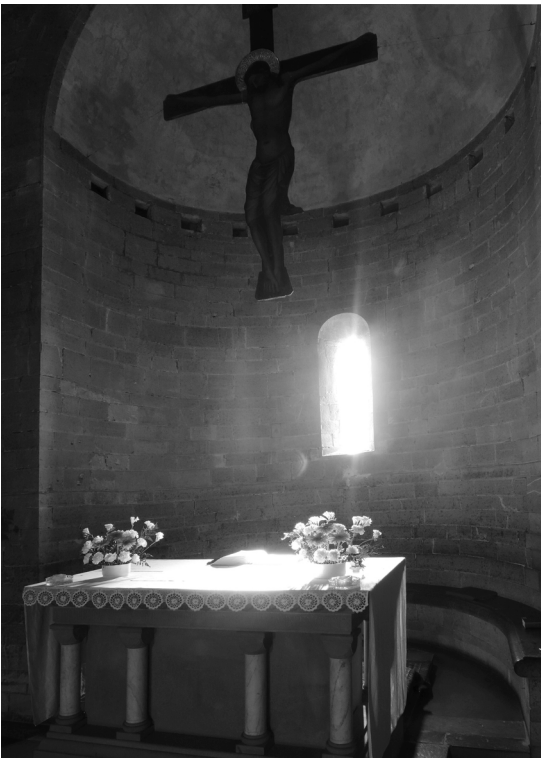


Fig. 4. Illumination of the altar in the church of Sant'Andrea a Mosciano (Scandicci, Florence). The church is oriented towards East-southeast, azimuth = 112.5° (picture: April 4, 8.20 a.m.).



Fig. 5. The altar of the Pieve di San Giovanni (Campiglia Marittima, Livorno) illuminated by the Easter sunlight. The church is oriented towards East-southeast, azimuth = 103.3° (picture: April 20, 8.20 a.m.).



Fig. 6. The Sun filtering through the window above the apse illuminates the counter-façade in the Abbey of Santi Salvatore e Cirino ad Abbadia Isola (Monteriggioni, Siena) at the sunrise of the winter solstice, azimuth = 127° (picture: December 21, 8.20 a.m.).

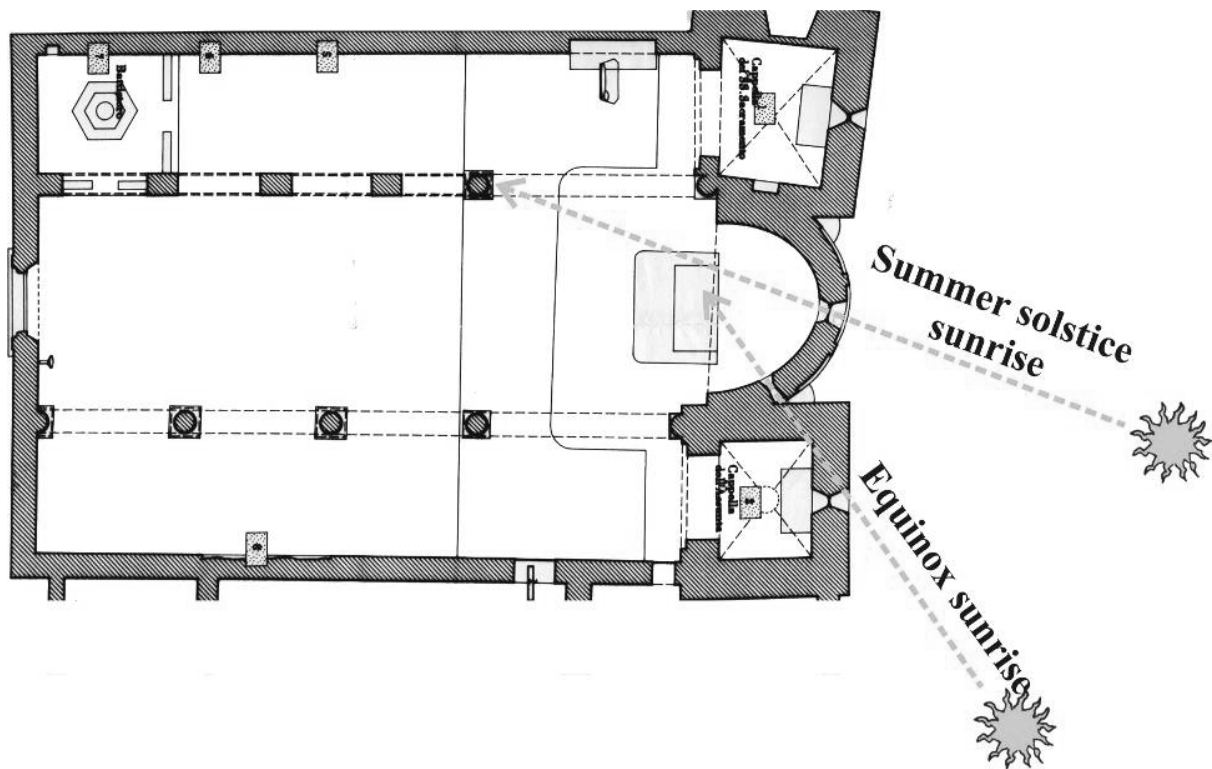


Fig. 7. Plan of the Pieve di Sant'Appiano a Barberino Val d'Elsa, azimuth = 35°; the single-lancet windows of the apse are shown. The bell tower on the right of the apse was built later and the equinoctial sunlight originally filtered through the window on the right.

Although nearly 20% of the churches we took into consideration do not present a clear Sun-related orientation, it is still possible at least to hypothesize a correlation with astronomical events. This is the case of the Pieve di Sant'Appiano (azimuth = 35°) because the single-lancet windows of the apse are oriented towards the sunrise of the summer solstice and towards the equinoctial sunrise, respectively (Figures 7 and 8). Furthermore, a solar symbolism is suggested in those churches which azimuth is comprised between 130° and 139° (16 out of 383, i.e. circa 4% of the churches examined) like in the case of the Basilica of San Miniato al Monte in Florence (azimuth = 131.5°) (Bartolini 2013), as we previously reported. In fact, on the winter solstice the Sun reaches an elevation angle between 3° and 11° with respect to the church axis so that the light can filter through the opening on the apse thereby illuminating the main entrance door. This phenomenon is observed in the Pieve di San Pietro a Cedda (Poggibonsi, Siena) (Figures 9 and 10) and in the Pieve di Cellole (San Gimignano, Siena) where the Sun creates a path of light that appears to conduct the devotees to the altar.

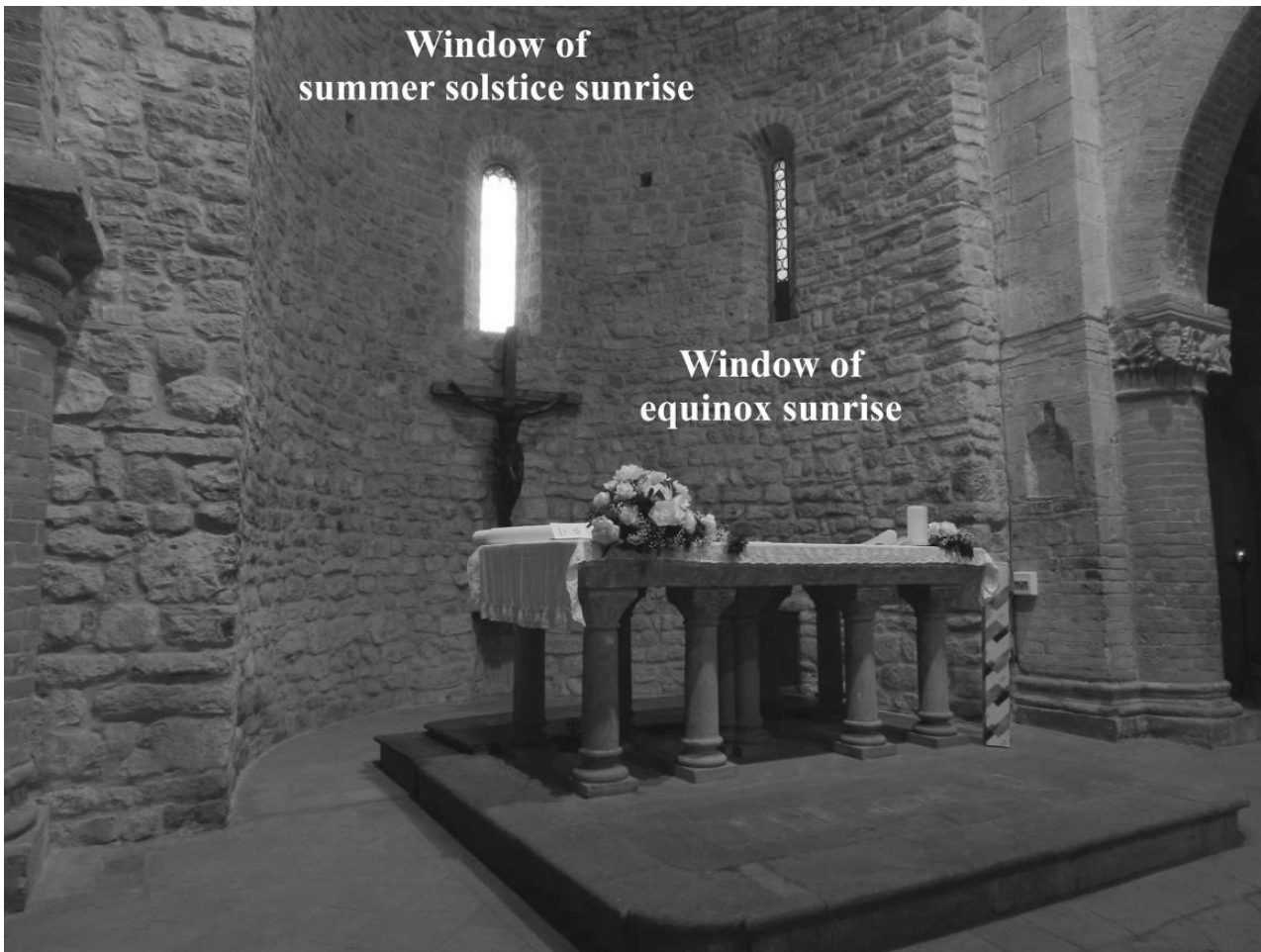


Fig. 8. The single-lancet windows of the apse of the Pieve di Sant'Appiano a Barberino Val d'Elsa.

In addition to that, a small group of churches present an azimuth that is comprised between 237° and 304° (18 out of 383, i.e. circa 5% of the churches). These buildings are oriented to the West, which means that the church entrance is directed to the East. Although we can speculate that some churches might have been rebuilt or restored, others, such as the Pieve di San Giovanni Battista a Mensano (Colle Val d'Elsa, Siena) and the Pieve di San Martino (Sesto Fiorentino, Florence), were actually projected with their façade towards the East, overturning the preminent criteria *Ad Orientem*. Perhaps, this orientation was intended to adopt the rule of the early Christian architecture, when sacred buildings were erected with the façade towards the sunrise, like in the case of the Old Saint Peter's Basilica; or, simply, those churches were built that way because of the urban structure. In the end, the grand total of churches with a clear or a possible relation with the Sun orientation is 344 (310+16+18) out of 383, corresponding to 90% circa of the churches we examined. We also noticed

that both large Basilicas in town centres and small churches in the countryside were erected following the same orientation criteria.



Fig. 9. The sunlight of the winter solstice filtering through the central opening of the apse (not shown) illuminates the main entrance of the church of San Pietro a Cedda (Siena), azimuth = 135°.

4. Discussion and conclusions

In this paper, we report the study on the orientation of 383 Romanesque churches located in the territory of the ancient dioceses of Tuscany based on their importance and on the conservation of original windows. The results obtained indicate that 81% of the churches (310 out of 383) we examined are oriented towards the local sunrise. In particular, we identified three main criteria for the orientation of sacred buildings, directed towards the solstitial sunrise, towards the equinoctial sunrise or the sunrise of the Easter period, or generally towards the local sunrise. In detail, we found out that:

- 1) 25 churches (10 + 15, i.e. 8.2% of 310 churches) are oriented towards the solstices. In particular, 15 of them are related to the sunrise of the winter solstice, *Sol Invictus*;
- 2) 122 churches (100 + 22, i.e. 39.2%) are oriented towards the sunrise of the Easter period or related to the *Sol Aequinoctialis*;

3) 163 churches (144 + 19, i.e. 52.6%) are oriented towards the East-southeast or the East -northeast, according to the criterion *Versus Solem Orientem*.



Fig. 10. The sunlight of the winter solstice filtering through the central opening of the apse illuminates the floor of the church of San Pietro a Cedda (Siena), creating a marvellous path of light that appears to take the devotees to the altar.

The position of the first group of churches is related to the solstices: these astronomical events were considered by the ancient to be the heaven's gates, through which the souls descended to Earth and ascended to Heaven (Champeaux, Sterckx 1992). Indeed, in the churches that are oriented towards the winter solstice (The gate of Gods) the light illuminates the main door at the sunrise, whereas in the churches directed towards the summer solstice (The gate of Man) the sunlight filtering through the openings on the façade illuminates the apse and the altar. Furthermore, we can hypothesize that illumination of the main door in the churches oriented towards the winter solstice symbolizes the enlightenment of the devotees during the period of Christmas.

In the second group, churches are built with the apse oriented towards the sunrise of the Easter period or the equinoctial sunrise. This orientation is linked with the Easter Sunday, when the light illuminating the apse

symbolizes the resurrection of Christ. In the case of churches oriented towards the equinoctial sunrise, there could be a connection with the *Sol Aequinoctialis*, whose rays illuminated the *Sancta Sanctorum* of the Solomon's Temple, the holy temple in ancient Jerusalem.

Finally, in the third group churches are directed to East-southeast or to East-northeast, *Versus Solem Orientem*. This orientation suggests a link with the direction of prayer, towards Jerusalem, which was a very common practice in medieval architecture. However, it is very probable that this orientation simply refers to the sunrise of the day of the foundation of the church: this would explain why nearly half of the churches has such a generic orientation. In fact, the architects had 9 months at their disposal to begin the construction works of the sacred building *Versus Solem Orientem*: 5 months for the churches oriented towards the East-southeast, 4 months for the churches oriented towards the East -northeast.

In addition to the 310 churches that present a clear Sun-related orientation, at least 34 (16 + 18) more sacred buildings present a possible relation with the Sun, because of the peculiar orientation of some windows that creates suggestive light effects. This is the case of the Pieve di San Pietro a Cedda (Poggibonsi, Siena) where the sunlight filtering through the opening on the apse illuminates the main entrance door and creates a path of light that appears to conduct the devotees to the altar.

In conclusion, this study dispels the commonly held notion that all the abbeys and churches erected during the Romanesque period in Tuscany are oriented towards the *Sol Aequinoctialis*; in fact, only 22 of the 383 churches we examined are oriented towards the equinoctial sunrise. However, a vast majority (90%) present a solar correlation: in fact, we found out that a grand total of 344 out of the 383 churches present a correlation with the Sun either because they are oriented towards the sunrise of a specific astronomical event, or because some windows are oriented in such a way that the sunlight can illuminate certain architectural elements. These criteria were applied in the construction of both large Basilicas in town centres and small churches in the countryside.

Certainly, this study is not exhaustive, since a lot of Romanesque churches in the ancient dioceses of Tuscany need to be examined, but it can provide a starting point to further investigate the possible solar correlation of the thousands of sacred buildings in Tuscany and beyond. Some churches could be oriented towards the sunrise of important festivities, such as the day of death or martyrdom of the Saint to which the church is entitled (Spinazzè 2015). Many light effects related to the Sun

have been hidden in churches for centuries, and only an enthusiastic and accurate examination can “shed new light” on this intriguing symbolism (Incerti 1999). This has been the case of the illumination of the sign of Cancer on the marble zodiac of San Miniato al Monte in Florence (Bartolini 2013; Bartolini, Pierozzi 2016), that, albeit it was under the eyes of millions of people, still remained “unrevealed” for 800 years.

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